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# Asymptotic Analysis of Gaussian Integrals, II: Manifold of Minimum Points

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**Abstract.** This paper derives the asymptotic expansions of a wide class of Gaussian function space integrals under the assumption that the minimum points of the action form a nondegenerate manifold. Such integrals play an important role in recent physics. This paper also proves limit theorems for related probability measures, analogous to the classical law of large numbers and central limit theorem.

## 1. Introduction

In the last few years, theoretical physicists have developed beautiful new ideas for the asymptotic analysis of Gaussian function space integrals [Coleman; Sect. 2], [Wiegel]. In this analysis one is confronted by the "zero mode problem". The object of this paper is to provide the mathematical framework for handling this problem. In particular, we present the complete asymptotic expansions of a class of Gaussian integrals on a Hilbert space, for which the minimum points of the action form a nondegenerate manifold. In addition, we prove limit theorems for related probability measures, analogous to the classical law of large numbers and central limit theorem.

To describe our problem, let $P_A$ denote a mean zero Gaussian probability measure with covariance operator $A$ on a real separable Hilbert space $\mathscr{H}$. We write the inner product of $\mathscr{H}$ as $\langle -, - \rangle$. We wish to describe the asymptotics of

$$J_n \doteq \int \psi(Y/\sqrt{n}) \exp(-nF(Y/\sqrt{n}))dP_A(Y), \quad \text{as } n \to \infty. \tag{1.1}$$

For simplicity we assume that $\psi, F$ are smooth functionals (smooth will always mean $C^\infty$), with $\psi$ bounded and $F$ satisfying

$$F(Y) \geqq -b\|Y\|^2 - c \text{ for some } 0 \leqq b < 1/(2\|A\|), \quad c \geqq 0. \tag{1.2}$$

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