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Linking Numbers, Contacts, and Mutual Inductances of a Random Set of Closed Curves

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Abstract. We establish here a new, general result of integral geometry, concerning closed rigid curves of arbitrary shapes in E^3 and their linking numbers I. It generalizes by a different method, the interesting integral property of I^2 found recently by Pohl and extended by des Cloizeaux and Ball, for two curves. We consider n closed curves linked successively to each other and forming a ring. The cyclic product of their linking numbers is integrated over the group of rigid motions of the curves. This integral is shown to factorize over a special algebra of characteristic functions. Each curve possesses two such intrinsic functions. The same algebra is shown to describe a larger class of integral geometry properties: a new theorem is established for a family of displacement integrals involving linking numbers, contact angles, and mutual inductances of the set of n curves.

1. Introduction

Topology of knotting and linking of closed curves in three-dimensional space involves very interesting mathematical problems. Two configurations of a set of closed curves are said to be topologically equivalent if one can transform continuously one into the other without forming any double point, (or opening one curve). The main problem is therefore to differentiate the irreducible configuration classes. This subject is also important in polymer theory. Polymer chains forming rings can indeed be considered ideally as closed solid curves. They are free to change their shape but not to open. Thus a given set of polymer rings may not develop new linkings by a simple continuous change of configuration. The statistical mechanics of a set of rings is then defined in a phase space restricted by topological constraints. Any progress in mathematical description of the linking of closed curves could be very useful.

Several topological invariants have been proposed for distinguishing whether or not two curves are linked together. In general these invariants do not yield a complete topological description. Two configurations with different values of

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