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Asymptotic Behaviour of the Classical Scalar Fields and Topological Charges

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Abstract. The existence and the properties of the limit at spatial infinity are studied for the finite-energy scalar fields with respect to the topological charge introduction. The limit is shown to be constant in time and in almost all spatial directions. The proof of the existence of the limit given by Parenti, Strocchi and Velo is extended to two-dimensional space. A generalized definition of the topological charge is suggested for a σ-model as an example.

1. Introduction and Conclusions

The possibility of introducing conserved topological charges [1] widely used in the soliton and instanton physics depends substantially on the asymptotic behaviour of the fields at spatial infinity. In the present paper, the existence and the properties of the limit at spatial infinity for a system of classical scalar fields with a finite energy are discussed from this point of view.

We consider a system of real scalar fields $\varphi : \mathbb{R}^s \times \mathbb{R} \to \mathbb{R}^n$,

$$\varphi(x,t) = \begin{pmatrix} \varphi_1(x,t) \\ \vdots \\ \varphi_n(x,t) \end{pmatrix}, \qquad x = (x^1, \ldots, x^s)$$

with continuous first derivatives [1]. The Lagrangian of the system is assumed to be of the form

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - U(\varphi)$$

leading to the field equations

$$\Box \varphi_j(x,t) + \frac{\partial}{\partial \varphi_j} U(\varphi(x,t)) = 0 \qquad (j = 1, \ldots, n)$$

1 The second derivatives of the fields and the first derivatives of the potential U appear in the field equations but most of the following statements are valid for the field configurations of finite energy regardless of field equations

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