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The Cauchy Problem for Coupled Yang–Mills and Scalar Fields in the Temporal Gauge

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Abstract. We study the Cauchy problem for minimally coupled classical Yang–Mills and scalar fields in $n+1$ dimensional space-time in the temporal gauge. We prove the existence and uniqueness of solutions for small time intervals and for any n. We then develop a general theory of solutions in local spaces and extend the previous local (in time) results to this more general setting. In space-time dimensions two and three, we prove the existence of global (in time) solutions by the method of a priori estimates, both in global and local spaces. In space-time dimension four, our estimates yield only partial results on the global existence problem.

1. Introduction

In spite of the large amount of work which has been devoted in the last years to the Yang–Mills equations, a satisfactory understanding of the properties of their solutions is still lacking. At the classical level most of the effort has gone into the Euclidean version of the theory, partly because of the richness of its geometrical structure, and partly because of its possible relevance to the quantized theory. On the other hand the theory in Minkowski space-time poses an interesting and non-trivial problem from the point of view of non-linear partial differential equations, and may appear as the classical limit of the quantized theory. The equations in Minkowski space-time have been studied most notably by Segal [11] who has considered the Cauchy problem for the pure Yang–Mills theory in four dimensional space-time, in the so-called temporal gauge $A^0 = 0$, giving a proof of the existence and uniqueness of solutions of the Cauchy problem for small time intervals. More recently, for coupled Yang–Mills and scalar fields in four dimensions, Glassey and Strauss have studied the time decay of solutions [4], [5] and have obtained a class of global solutions of a special type suggested by the known

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