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The Kosterlitz-Thouless Transition in Two-Dimensional Abelian Spin Systems and the Coulomb Gas

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Abstract. We rigorously establish the existence of a Kosterlitz-Thouless transition in the rotator, the Villain, the solid-on-solid, and the $\mathbb{Z}_n$ models, for n large enough, and in the Coulomb lattice gas, in two dimensions. Our proof is based on an inductive expansion of the Coulomb gas in the sine-Gordon representation, extending over all possible distance scales, which expresses that gas as a convex superposition of dilute gases of neutral molecules whose activities are small if β is sufficiently large. Such gases are known not to exhibit screening. Abelian spin systems are related to a Coulomb gas by means of a duality transformation.

1. Introduction

1.1. General Remarks

In this paper we rigorously establish the Kosterlitz-Thouless transition [1] in a class of two dimensional models including the plane rotator, the $\mathbb{Z}_n$-model for n sufficiently large and the lattice Coulomb gas. These results and a brief sketch of the proof have already appeared in [2]. Our methods extend to higher dimensional abelian spin systems, abelian lattice gauge theories and to the one-dimensional Ising model with $1/r^2$ interaction. Details of these extensions will appear elsewhere.

All the models we shall analyze are known to have a high temperature phase with exponentially decaying truncated correlations. For example, the Coulomb gas has a high temperature, low density plasma phase characterized by exponential Debye screening [3]. The Kosterlitz-Thouless transition is one from a high temperature phase to a low temperature phase characterized by scaling and a power law fall-off of correlations. In this paper we shall prove the following results:

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