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The Riemannian Geometry of the Configuration Space of Gauge Theories

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Abstract. We state some new results about the configuration space of pure Yang-Mills theory. These results come from the study of the kinetic energy term of the Lagrangian of the theory. This term defines a riemannian metric on the space of non-equivalent gauge potentials. We develop a riemannian calculus on the configuration space, compute the riemannian connection, the curvature tensor, and solve for the geodesics, etc. We show that the Gribov ambiguity is more than an artefact of the choice of a gauge condition, and is related to the existence of conjugate points on the geodesics, and is thus an intrinsic feature of the theory.

Introduction

Pure Yang-Mills theory yields one of the most interesting examples of a singular Lagrangian in field theory (i.e. in a system with an infinite number of degrees of freedom). A serious analysis of this system requires the use of Dirac's formalism for systems with constraints [1–3].

Our analysis differs from that of [2] in that, instead of introducing second class constraints to eliminate the gauge freedom, we directly define the theory on the space of non-equivalent gauge potentials (quotient space by the action of the group of gauge transformations, which we call orbit space). This analysis leads us to the definition of the *true* configuration space $\mathfrak{M}$ of the theory, and to an effective Lagrangian defined on $\mathfrak{M}$. This Lagrangian contains a kinetic part (the electric part) and a velocity independent potential term (the magnetic part). The kinetic term provides a riemannian metric on $\mathfrak{M}$ by saying that it is of the form $\frac{1}{2}\cdot\{$square of the velocity computed with that metric$\}$. Consequently the classical motion is the motion of a point in an infinite dimensional riemannian manifold in a potential.

The study of the configuration space is essential, first because it is the space of classical physical states of the system, secondly because the quantum functional integral is to be defined on paths on this space. The metric defined on $\mathfrak{M}$ is a

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