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Capacity and Quantum Mechanical Tunneling

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Abstract. We connect the notion of capacity of sets in the theory of symmetric Markov process and Dirichlet forms with the notion of tunneling through the boundary of sets in quantum mechanics. In particular we show that for diffusion processes the notion appropriate to a boundary without tunneling is more refined than simply capacity zero. We also discuss several examples in $\mathbb{R}^d$.

1. Introduction

In recent years the theory of symmetric Markov processes has been developed considerably using the connection with the theory of Dirichlet forms, see [1] and references therein. In particular the case of processes over an open set in $\mathbb{R}^d$, given by local generators with singular coefficients (that need not be functions nor generalized functions) has been studied. The usefulness of this approach for quantum theory has been pointed out in [2–5, 29]. It permits the definition of quantum dynamics in situations where the approach via a potential perturbing a kinetic energy term does not work, due to the singularities of the potential or the fact that the potential is neither a measurable nor a generalized function, like in cases of zero range potentials considered in nuclear physics and solid state physics (see e.g. [12–14], and references therein). This is an extension of the usual approach to the definition of Hamiltonian inasmuch as in the cases where the Hamiltonian can be defined as a sum of a kinetic energy term and a not too singular potential, the approach by Dirichlet forms is equivalent with the traditional one [15, 6, 7, 2, 3, 8–10]. Another advantage of the approach to dynamics via Dirichlet forms is the fact that it extends to the case of infinitely many degrees of freedom (see [6, 3, 16] and references therein; see also [17–20]). Moreover it gives an immediate connection with the theory of diffusion processes, the Hamiltonian appearing as the infinitesimal generators of such processes, therefore making available for quantum theory results of this very well studied chapter of modern probability theory. In the other direction this connection suggests developments in the theory of diffusion processes. This connection can be

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