Commun. Math. Phys. 81, 495–499 (1981)

Communications in
Mathematical
Physics

A Poisson Random Walk is Bernoulli*

Steven Kalikow
Department of Mathematics, Stanford University, Stanford, CA 94305, USA

Abstract. Particles distributed on the integers in Poisson distribution, each independently taking a random walk, form a stationary Markov chain. The canonical shift in this space is Bernoulli.

1. Introduction

Consider particles distributed on $\mathbb{Z}$ with Poisson distribution, parameter 1. (That is for each $n \in \mathbb{Z}$, there is a nonnegative number ω_n of particles on n. The values $\{\omega_n\}_{n\in\mathbb{Z}}$ are independent identically distributed (i.i.d.) random variables, each with Poisson distribution parameter 1.) Let each particle take a random walk, described as follows. (Results can be shown to remain valid for any random walk but we choose this one for simplicity. A discussion of generalizing this result to arbitrary random walk will be made in a closing remark at the end of this paper.) We let each particle stand still, move one step forward, or one step backward, each with probability 1/3. They all move independently. Then the particles will remain Poisson distributed. By continuing this procedure we obtain a sequence of configurations $\{X_i\}_{i\in\mathbb{N}}$, where each X_i is a Poisson-distributed Ω-valued random variable, where $\Omega = \mathbb{N}^{\mathbb{Z}}$, $\mathbb{N} = \{0, 1, 2, \ldots\}$. Since $\{X_i\}_{i\in\mathbb{N}}$ forms a stationary Markov chain, we can extend the process to a stationary Markov chain $\{X_i\}_{i\in\mathbb{Z}}$. This process will henceforth be called Poisson Random Walk, abbreviated P-walk. The shift $\{X_i\} \to \{Y_i\}$, $Y_i = X_{i+1}$ for all i, is a stationary shift. Here it is shown to be a Bernoulli shift.

My result can easily be extended to a process where particles are labeled by their past and future, thereby strengthening a result of Goldstein and Lebowitz (Theorem 5.3, page 10 of [1]).

2. Probabilistic Statement of Theorem to be Proved

Let $\{X_i\}_{i\in\mathbb{Z}}$ be as in the Introduction. The theorem proved by this paper is

Theorem 1. *$\{X_i\}_{i\in\mathbb{Z}}$ is Bernoulli.*

* This research was supported in part by NSF grant MCS 78–07739–A03

0010-3616/81/0081/0495/$01.00