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Percolation for Low Energy Clusters and Discrete Symmetry Breaking in Classical Spin Systems

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Abstract. We consider classical lattice systems in two or more dimensions with general state space and with short-range interactions. It is shown that percolation is a general feature of these systems: If the temperature is sufficiently low, then almost surely with respect to some equilibrium state there is an infinite cluster of spins trying to form a ground state. For systems having several stable sets of symmetry-related ground states we show that at low temperatures spontaneous symmetry breaking occurs because in a two-dimensional subsystem there is a unique infinite cluster of this type.

1. Introduction

Percolation theory has recently been successfully used in analyzing the properties of the two-dimensional Ising model [1, 12]. Percolation is also implicitly involved in the well-known Peierls argument for the existence of phase transitions. Here we introduce the notion of "low energy clusters". This concept enables us to show that percolation at low temperatures is a general phenomenon for spin systems in two or more dimensions. Under certain circumstances this kind of percolation automatically implies the existence of several distinct equilibrium states; this gives an intuitively appealing picture of the mechanism which leads to a spontaneous breaking of discrete symmetries. (This picture fails to explain certain phenomena in three dimensions such as the breaking of continuous symmetries and the existence of nontranslation invariant Gibbs states in the Ising model.)

Passing on to precise definitions, let $d \geqq 2, L = \mathbb{Z}^d$ (the integer lattice), and consider a system of spins on L taking values in a probability space $(E, \mathscr{E}, \lambda)$. The spins interact via a nearest-neighbour potential which is a measurable symmetric function

$$\varphi : E \times E \to \mathbb{R},$$

satisfying

$$m \equiv \inf \varphi(\cdot,\cdot) > -\infty. \tag{1.1}$$

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