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Outer Automorphisms and Reduced Crossed Products of Simple C^*-Algebras

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Abstract. Every outer automorphism of a separable simple C^*-Algebra is shown to have a pure state which is mapped into an inequivalent state under this automorphism. The reduced crossed product of a simple C^*-algebra by a discrete group of outer automorphisms is shown to be simple.

0. Introduction

We consider two problems both of which depend on one technical lemma.

One of them is the problem, stated by Lance in [4], whether or not any universally weakly inner automorphism of a simple C^*-algebra is inner. In Sect. 2 we shall answer this affirmatively in case the C^*-algebra is separable. The idea of the proof is based on [3], the corresponding result in case of one-parameter automorphism groups.

The other is the problem whether or not the reduced crossed product of a simple C^*-algebra by a discrete group of outer automorphisms is simple. In Sect. 3 we shall answer this affirmatively. The proof is essentially the same as that of the result which has been obtained by Elliott in [1] in case the C^*-algebra is AF (i.e., approximately finite-dimensional).

In Sect. 1 we shall give a main lemma on outer automorphisms; a similar result has also been obtained by Elliott in the AF case.

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1. Outer Automorphisms

Let A be a C^*-algebra and α an automorphism of A. In [2] we have defined the strong Connes spectrum $\tilde{\mathbb{T}}(\alpha)$ of α. In this case $\tilde{\mathbb{T}}(\alpha)$ is the set of $\lambda \in \mathbb{T} = \hat{\mathbb{Z}}$ such that $\hat{\alpha}_\lambda(I) = I$ for any primitive ideal I of the crossed product $A \times_\alpha \mathbb{Z}$ of A by α, where $\hat{\alpha}$ is the dual action. $\tilde{\mathbb{T}}(\alpha)$ depends on α up to inner automorphisms, i.e., $\tilde{\mathbb{T}}(\operatorname{Ad} u \circ \alpha) = \tilde{\mathbb{T}}(\alpha)$, where u is a unitary multiplier of A. If A is α-simple, i.e., A does not have

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