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Translationally Invariant States and the Spectrum Ideal in the Algebra of Test Functions for Quantum Fields

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Abstract. A class of states on Borchers' tensor algebra is constructed. These states are invariant under the translation group and fulfill the spectrum condition. This leads to a characterization of the linear span of all such states in terms of a simple continuity property.

1. Introduction

In the algebraic formulation of Wightman's axioms, quantum fields correspond to a class of positive, linear functionals on a topological $*$-algebra, the tensor algebra over a space of test functions [1 – 3]. The conditions which distinguish Wightman functionals from other positive functionals on the algebra are invariance under a group of automorphisms and the requirement that the functionals vanish on two prescribed ideals. Various general aspects of the positive linear functionals on this algebra have been studied, e.g. in [4–14], cf. also [15] and literature quoted therein. The invariance condition and the two ideals play almost no role in these works, however. In the present paper, we want to take into account a part of the automorphism group, the translations of space-time, and one of the ideals, which corresponds to the spectrum condition for energy and momentum. We give a characterization of the linear span of all positive, invariant functionals satisfying a general spectrum condition. From this characterization follows in particular, that the invariant positive functionals span a dense subspace of the space of all invariant functionals and that the spectrum ideal is the intersection of the left kernels of the positive invariant functionals which annihilate it.

It is perhaps worthwhile to point out some differences between the present framework and the theory of C^*-algebras. On a C^*-algebra, there are just as many invariant states for a given group of automorphisms as there are invariant linear functionals. This is so because the Jordan decomposition of a linear functional automatically preserves invariance. Moreover, for an amenable group of automorphisms, one can construct an invariant functional from an arbitrary functional by using an invariant mean. For the tensor algebra these methods do not work. First,

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