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On the Absence of Spontaneous Symmetry Breaking and of Crystalline Ordering in Two-Dimensional Systems

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Abstract. We develop a unified approach, based on Araki's relative entropy concept, to proving absence of spontaneous breaking of continuous, internal symmetries and translation invariance in two-dimensional statistical-mechanical systems. More precisely, we show that, under rather general assumptions on the interactions, all equilibrium states of a two-dimensional system have all the symmetries, compact internal and spatial, of the dynamics, except possibly rotation invariance. (Rotation invariance remains unbroken if connected correlations decay more rapidly than the inverse square distance.) We also prove that two-dimensional systems with a non-compact internal symmetry group, like anharmonic crystals, typically do not have Gibbs states.

1. Introduction and Main Results

It is well known that continuous symmetries of two-dimensional statistical mechanical systems or two-space-time-dimensional quantum field theories cannot, in general, be broken spontaneously (except in systems with interactions of very long range). Mathematical proofs of this fact have been known for quite a long time: They have appeared in work of Mermin and Wagner [1] concerning quantum spin systems on a two-dimensional lattice, of Mermin [2] concerning classical lattice spin systems, and in [3] where classical particle systems have been analyzed. For related results concerning quantum field theory, see [4], [5]. In [1] and [2] it is shown that the spontaneous magnetization vanishes and in [3] that the density of particles is constant, thus excluding the existence of crystalline order. Physical background material as well as the mathematical outline of the proofs are very well explained in [6]. The basic tool is Bogoliubov's inequality, which was used for the first time in this context by Hohenberg in his study of the Bose gas [7]. (A rigorous proof was later published in [8].) Using Bogoliubov's inequality, Fisher and Jasnow [9] proved clustering properties of the two-point function and, consequently, that the order parameter vanishes. McBryan and Spencer obtained a better decay for the two-

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