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Some Twisted Self-Dual Solutions for the Yang-Mills Equations on a Hypertorus*

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Abstract. The $SU(N)$ Yang-Mills equations are considered in a four-dimensional Euclidean box with periodic boundary conditions (hypertorus). Gauge-invariant twists can be introduced in these boundary conditions, to be labeled with integers $n_{\mu\nu}(=-n_{\nu\mu})$, defined modulo N. The Pontryagin number in this space is often fractional. Whenever this number is zero there are solutions to the equations $G_{\mu\nu}=0$. Here $G_{\mu\nu}$ is the covariant curl. When this number is not zero we find a set of solutions to the equations $G_{\mu\nu}=\tilde{G}_{\mu\nu}$, provided that the periods a_μ of the box satisfy certain relations.

1. Introduction

Understanding quantized gauge theories in the strong-interaction region is made difficult by severe infrared divergences. It is therefore useful to consider gauge models enclosed in a box with sides of variable lengths. As for the boundary conditions at the sides periodic boundary conditions are the most natural choice [1]. Indeed, computer simulations have been made of gauge theories in such boxes and taught us much about their phase structure [2].

After having dealt with the vacuum in the box one may consider studying some of the first excited states, such as those corresponding to a hadronic particle trapped in the box. But it is perhaps of more fundamental importance to look at a trapped amount of electric or magnetic flux in the box. The first of these would correspond to a string connecting two opposite sides of the box. (This is the string which in the infinite volume limit is believed to confine quarks inside hadrons.) The energy of such a state corresponds directly to the string constant. In [1] it is explained how this state is described in terms of field configurations in a box where the periodic boundary conditions have

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