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# A Note on the Power Spectrum of the Iterates of Feigenbaum's Function*

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**Abstract.** We give rigorous bounds on the scaling behaviour of the power spectrum for Feigenbaum's map.

Recently, estimates have been given concerning the scaling behaviour of the power spectrum for the Feigenbaum map, Feigenbaum [1, 2], Nauenberg and Rudnick [3]. Since these estimates are based on somewhat heuristic arguments, it might be useful to give *rigorous bounds* on this scaling behavior. The purpose of this note is to give such bounds.

We denote by $\varphi$ the Feigenbaum function, defined [4] by the relations

$$\varphi(\varphi(\lambda x)) = -\lambda\varphi(x), \qquad \lambda = -\varphi(1), \qquad \varphi(0) = 1, \tag{1}$$

$\varphi$ analytic. For every diadic rational $q = k/2^N$, $k$ odd, we define the amplitudes

$$a(q) = \lim_{p\to\infty} \frac{1}{p} \sum_{j=0}^{p-1} \exp(2\pi i\, kj/2^N)\, \varphi^j(0).^1 \tag{2}$$

(The limit exists since $\{\varphi^j(0)\}$ is almost periodic.) We define the averaged square amplitude $A_N$ at level $N$ as

$$A_N = \frac{1}{2^{N-1}} \sum_{k=0}^{2^{N-1}-1} |a((2k+1)/2^N)|^2. \tag{3}$$

**Theorem.** *For all $M > 0$, one has the bound*

$$\left(\frac{\lambda^2}{4}\right)^M \leqq \frac{A_{N+M}}{A_N} < \left(\frac{\lambda^2}{4}\right)^M X Y^M,$$

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[1] $\varphi^j(x) = \varphi(\varphi^{j-1}(x)), \quad j > 0, \quad \varphi^0(x) = x$

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