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Propagation of States in Dilation Analytic Potentials and Asymptotic Completeness

Peter A. Perry*

Department of Physics, Princeton University, Princeton, NJ 08544, USA

Abstract. We estimate the space-time behavior of scattering states for two-body Schrödinger operators with smooth, dilation analytic potentials. We use our estimates to give a simple proof of asymptotic completeness for a class of long-range potentials, including the Coulomb potential plus a fairly general short-range perturbation.

Introduction

The goal of this paper is to present a simple proof of asymptotic completeness for the modified wave operators that describe two-body quantum scattering with certain long-range potentials. Modified wave operators were introduced by Dollard [6] to study scattering for the Coulomb potential. Spectral and scattering theory for general long-range potentials has since been studied by many authors. Spectral representations for such long-range Schrödinger operators have been studied by Ikebe [14, 15] and Saitō [31, 32]. Their results imply completeness of the stationary wave operators defined via the spectral representation. Isozaki [18] proved completeness of the stationary wave operator and Kitada [22–24] proved completeness of time-dependent modified wave operators by a stationary method. More recently Ikebe and Isozaki [16, 17] have also given a proof of completeness for the modified wave operators. Agmon [1] has also proved completeness results for Schrödinger operators with long-range potentials and Enss [8] has given a "geometric" proof of completeness for certain long-range potentials.

Here we would like to give a simple, "geometric" proof of completeness for Schrödinger operators $H_1 = H_0 + V + \overline{V}$ on $L^2(\mathbb{R}^n)$, where $H_0 = -\frac{1}{2}\Delta$, V is a long-range, dilation analytic potential, and $\overline{V}$ is a fairly general short range perturbation (we formulate precise hypotheses below). Our class of potentials thus includes the Coulomb potential plus a fairly general short-range perturbation. Our assumptions

* Present address: Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, NY 10012, USA

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