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Lax Representation for the Systems of S. Kowalevskaya Type

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Abstract. We describe a number of dynamical systems that are generalizations of the S. Kowalevskaya system and admit the Lax representation.

It is well known (see for example [1, 2]), that the equations of motion of a three-dimensional heavy rigid body rotated about a fixed point is a completely integrable dynamical system only in the cases of Euler [3], Lagrange [4], and S. Kowalevskaya [5]. In the present note we describe a number of systems that are of the Kowalevskaya type and admit the Lax representation[1].

1. Let $\mathscr{G}$ be the Lie algebra of a group G, $\mathscr{G}^*$ be the space dual to $\mathscr{G}$, $\{x_\alpha\}$ be the coordinates of a point in the space $\mathscr{G}^*$. In the space $\mathscr{F}(\mathscr{G}^*)$ of smooth functions on $\mathscr{G}^*$, let us define the Poisson bracket[2]

$$\{f, g\} = C^\gamma_{\alpha\beta} x_\gamma \partial^\alpha f \partial^\beta g, \qquad \partial^\alpha = \partial/\partial x_\alpha. \tag{1}$$

Here $C^\gamma_{\alpha\beta}$ are the structure constants of the Lie algebra. The space $\mathscr{F}(\mathscr{G}^*)$ is endowed by the above formula with the structure of the Lie algebra. A dynamical system in $\mathscr{G}^*$ is determined by a Hamiltonian function $H(x) \in \mathscr{F}$, so that the equations of motion have the form

$$\dot{x}_\alpha = \{H, x_\alpha\}. \tag{2}$$

The coadjoint representation of the group G acts on the space $\mathscr{G}^*$. Orbits of this representation are invariant with respect to an arbitrary Hamiltonian H, and are the phase spaces of the considered systems.

2. Let G be a compact simple Lie group, K be its subgroup such that the factor space G/K is a symmetric space [6]. Then $\mathscr{G} = \mathscr{K} \oplus \mathscr{S}$, $\mathscr{K}$ is the Lie al-

[1] Note that the Lax representation for equations of motion of a rigid body was discovered first by S. Manakov [8] for some particular cases

[2] In this note we use the tensor notations; in particular, everywhere repeated indices imply summation

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