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Some Relations Between Dimension and Lyapounov Exponents

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Abstract. We consider differentiable maps and compact invariant sets. We introduce dimensional quantities related to the ergodic invariant measures, and prove some simple relations.

We consider differentiable maps and compact invariant sets. An estimate from above for the Hausdorff dimension of such a set has been given by A. Douady and J. Oesterlé [DO] and by Mañe [M_1]. In this paper we discuss some other relations of this kind. We first show how to deduce an estimate involving Lyapunov exponents of the system. We also introduce the fractal dimension $f(m)$ of a measure m on a compact space, which weights an "essential" dimension of (X, m).

The results are the following: for any ergodic invariant probability measure, we consider the spectrum of the linear tangent map (the so-called Lyapunov exponents) and the "dilating dimension" of this spectrum dim dil Sp m; the dimension of a compact invariant set is bounded from above by the supremum of dim dil Sp m over all invariant probability measures; individually, for any ergodic invariant probability measure, we have

$$f(m) \leqq \dim \operatorname{dil} \operatorname{Sp} m.$$

This inequality is generally a strict inequality, as is shown by considering maps of the interval, where $f(m)$ is related to the entropy $h(m)$ and the positive Lyapunov coefficient λ by $f(m) = \frac{h(m)}{\lambda}$.

This notion of dimension of a measure is closer to what is actually measured in experiments like those performed by P. Frederikson, J. Kaplan and J. Yorke [FKY]. It leads us to reformulate these conjectures there and to discuss some other questions.

I. Notations and Results

Let L be a linear operator from an euclidean space E of dimension d in an euclidean space F. Define s-numbers of L, denoted $[L]$, as the decreasing sequence

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