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Cancellations of Infrared Divergences in the Two-Dimensional Non-Linear σ Models

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Abstract. In the two-dimensional $O(N)$ nonlinear σ models, the expectation value of any $O(N)$ invariant observable is shown to have an infrared finite weak coupling perturbative expansion, although it is computed in the "wrong" spontaneously broken symmetry phase. This result is proved by extracting all infrared divergences of any bare Feynman amplitude at $D = 2 - \varepsilon$ dimension. The divergences cancel at any order only for invariant observables. The renormalization at $D = 2$ preserves the infrared finiteness of the theory.

1. Introduction

Two-dimensional σ-models have raised an increasing interest during the last years, owing to their similarity with four-dimensional gauge theories, their simpler structure and the development of powerful nonperturbative methods. In this paper we deal with the weak coupling perturbative approach. This approach suffers from the fact that the naive vacuum state is not the true one, as presumably is the case for four-dimensional gauge theories. Indeed, the perturbative expansion has to be made around a peculiar classical solution, i.e. in the spontaneously broken symmetry phase, although such a phase cannot exist in two-dimensional space [1, 2]. So the symmetry has to be dynamically restored for any positive coupling constant [3, 4, 5]. A drastic consequence of the fact that the perturbative expansion is made in the wrong phase is that this expansion has very important infrared divergences, since even the free propagator of a massless Goldstone boson does not exist at two dimensions. For this reason the first perturbative calculations have been performed by introducing a symmetry breaking term which makes the theory infrared (I.R.) finite (and then by setting this term to zero) [4, 5].

However, S. Elitzur, following a remark of A. Jevicki about the effective potential of the $O(N)$ σ model [6], conjectured that any $O(N)$ invariant observable has an infrared finite expectation value to any order in perturbation expansion [7], and checked the fact up to third order of the two-point function. Various computations have been made by some authors [8, 9] for the $O(N)$ and $G \otimes G$ chiral models which have verified the conjecture in many cases and used it to study these models. Moreover, this result is very similar to what is expected for four-dimensional gauge theories, a namely that some gauge invariant quantities should be

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