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Nonequilibrium Measures Which Exhibit a Temperature Gradient: Study of a Model

A. Galves[1]*, C. Kipnis[2]**, C. Marchioro[3], and E. Presutti[4]

1 Instituto de Matématica e Estatistica, Universidade de São Paulo, São Paulo, Brazil
2 Centre de Mathématiques de l'Ecole Polytechnique, Plateau de Palaiseau, F-91128 Palaiseau Cedex, France
3 Dipartimento di Matematica, Libera Università di Trento, I-38050 Povo (Trento), Italy
4 Istituto Matematico, Università di Roma, I-00100 Roma, Italy

Abstract. We give some rules to define measures which could describe heat flow in homogeneous crystals. We then study a particular model which is explicitly solvable: the one dimensional nearest neighborhood Ising model. We analyze two cases. In the first one the spins at the two boundaries interact with reservoirs at different temperatures; in the thermodynamical limit the measure we introduce converges locally to Gibbs measures and a temperature profile is so derived. We obtain an explicit expression for the thermal conductivity coefficient which depends on the temperature. In the second case we study the asymptotic behavior starting from an initial state in which each half of the space is at a different temperature. We find again a temperature profile which asymptotically obeys the heat equation with the thermal conductivity coefficient previously derived. From a mathematical point of view, the analysis of the invariant measure is made possible by studying a "time-reversed" process related to a graphical representation of an associated process. This provides us with an explicit formula for the n-fold correlation function and we study the limiting behavior using both this representation (for proving an exchangeability result) and a Donsker-type, spacetime renormalization procedure.

1. Introduction

In this paper we study the stationary nonequilibrium measures which describe systems where a temperature gradient is present and a heat flow is established. Even though this is a classical problem in Statistical Mechanics, many questions still remain unanswered, particularly in the framework of a mathematically rigorous approach. The main point of investigation [1, 2] concerns both a general characterization of these measures and the (dynamical) way the stationary heat flow is established. In this paper we restrict ourselves to the first aspect so as to

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