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Iterated Mayer Expansion for Classical Gases at Low Temperatures*

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Abstract. We derive iterated Mayer expansions for classical gases and establish recursive bounds which control their convergence. These bounds are useful for gases with two body forces which are strong and possibly attractive at distances that are short compared to their range. Our procedure is based on splitting the potential into pieces of decreasing strength and increasing range. This may be called a renormalization group treatment of a classical gas. We apply our results to Yukawa lattice gas models and obtain convergence of series expansions for the pressure for a range of parameters (temperature, fugacities, range of the interaction) that was inaccessible before. Application to 3-dimensional $U(1)$ lattice gauge theory (Coulomb gas, Z-ferromagnet) will be made elsewhere.

1. Introduction

In this paper we present some tools for the investigation of convergence properties of (generalized) Mayer expansions for classical gases [1]. They are useful when the particles of the gas interact through two body forces that are strong and possibly attractive at distances that are short compared to their range.

To be specific, consider a gas of particles with charge $m = \pm 1$ which can occupy the sites x of a cubic 3-dimensional lattice Λ with lattice spacing set equal to 1, and which interact through a Yukawa potential (= infrared cutoff Coulomb potential) of range M^{-1}. The Yukawa potential $v(xy) = (-\Delta + M^2)^{-1}(xy)$ is the translation invariant solution of the finite difference equation on $\mathbf{Z}^3$,

$$(-\Delta + M^2)v(xy) = \delta_{xy}. \tag{1.1}$$

Δ is the lattice Laplacian, viz. $\Delta f(x) = \sum_y (f(y) - f(x))$ (sum over nearest neighbours of x in $\mathbf{Z}^3$).

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