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Integral Representation for the Dimensionally Renormalized Feynman Amplitude

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Abstract. A compact convergent integral representation for dimensionally renormalized Feynman amplitudes is explicitly constructed. The subtracted integrand is expressed as a distribution in the Schwinger α-parametric space, and is obtained by applying upon the bare integrand a new subtraction operator R' which respects Zimmermann's forest structure.

1. Introduction

Dimensional renormalization [1–9], first introduced by Speer and Westwater [1] and applied in the study of gauge theories by t'Hooft and Veltman [3], has proved to be an essential tool in quantum field theory. Indeed, it preserves gauge invariance, Lorentz invariance and avoids the infrared problem which appears when subtractions are performed at zeromomenta. Another advantage of this renormalization is that the Callan-Symanzik [10] function $\beta(g)$ is then independent of the dimension of space time (apart from a trivial $(D-4)\,g$ factor) and is also independent of the mass ratios which enter the theory.

In recent years, dimensional regularization and dimensional renormalization were established on firm ground as were other kinds of renormalization, and we refer to the literature [7, 9, 11]. According to Bogoliubov-Parasiuk-Hepp (BPH) recurrence, the usual method to calculate such an amplitude is first to renormalize the smaller divergent subgraphs by extracting their poles at $D=4$, then to introduce their finite parts into larger subgraphs and reproduce the same procedure in a recurrent way. This method becomes very difficult at high orders of perturbation, dealing with overlapping divergences, spinor, coupling derivatives and gluon propagators.

On the other hand, the existence of a compact expression which, for a given Feynman graph, gives directly the dimensionally renormalized integrand is still missing. Some authors in the study of the properties of dimensional renormalization come close to achieving this goal (for instance, the C_H operators of Breitenlohner and Maison [7] or the $\mathscr{P}\mathscr{P}_\lambda$ operators of Collins [6], organized in forests of divergent subgraphs). But the successive applications of these operators,

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