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An n Monopole Solution with $4n-1$ Degrees of Freedom

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Abstract. An exact static monopole solution, possessing n units of magnetic charge and $(4n-1)$ degrees of freedom, is constructed, generalising the recent work of Ward on two monopole solutions. The equations solved are those of an $SU(2)$ gauge theory with adjoint representation Higgs field in the (BPS) limit of vanishing Higgs potential. The number of degrees of freedom is maximal for self-dual solutions. The construction is described in a deductive way, within the framework of the Atiyah-Ward formalism for self-dual gauge fields.

1. Introduction

Gauge field theories in which the symmetry group G is spontaneously broken, by the agency of a Higgs field in the adjoint representation, possess classical solutions with the natural interpretation of magnetic monopoles [1, 2]. (For a review see e.g. [3]). The magnetic charge of these solutions is quantised in that, for topological reasons, it has to be an integral multiple of 4π, in suitable units. We shall call a solution with magnetic charge $4\pi n$ an n monopole solution. In the limit in which the potential describing the self-interaction of the Higgs field vanishes, the Bogomol'nyi-Prasad-Sommerfield (BPS.) limit [4, 5], it is possible to produce some exact static finite energy solutions of the equations of motion, in terms of elementary functions. The first example, a charge one $SU(2)$ monopole, was spherically symmetric [5]. This has been generalised to obtain spherically symmetric solutions for larger gauge groups [6]. Recently, following a paper in which Ward constructed an axis symmetric two monopole solution [7], axis symmetric solutions of arbitrary charge have been proposed [8]. Further Ward has now produced a reasonably general solution of charge two [9]. In this paper we extend Ward's result to higher charge, analysing the construction in a way which we hope makes it appear rather natural.

In the BPS. limit the equations of motion are implied by the Bogomol'nyi equations

$$\mathbf{B}_i = \pm D_i \Phi, \tag{1.1}$$

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