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Ansätze for Self-Dual Yang-Mills Fields

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Abstract. A sequence $\mathscr{A}_1$, $\mathscr{A}_2$, ... of ansätze for generating self-dual solutions of the Yang-Mills equations is presented. For each n, $\mathscr{A}_n$ produces a solution depending on two arbitrary functions of three variables. As an application, we see that $\mathscr{A}_2$ generates a static Yang-Mills-Higgs 2-monopole solution.

1. Introduction

In recent years, there has been considerable interest in self-dual $SU(2)$ Yang-Mills fields in Euclidean space $\mathbb{R}^4$. In the first place, they arise as instantons, which dominate the Euclidean functional integral [1–3]. Secondly, they include, as a special case, static Yang-Mills-Higgs fields in space-time, in the Prasad-Sommerfield limit; these have come to be known as multi-monopoles [4–8]. One of the more successful ways of understanding the self-duality equation, and of generating solutions to it, has been the approach which arises out of Penrose's twistor theory [9]. This led to a sequence $\mathscr{A}_1, \mathscr{A}_2, \ldots$ of ansätze which generate all instanton solutions [10, 11]; and led also the Atiyah-Hitchin-Drinfeld-Manin (AHDM) construction [3] which generates the instantons even more effectively. More recently, these ansätze have been used to construct multi-monopole solutions of the Yang-Mills-Higgs-Bogomolny (YMHB) equations [7, 8].

The purpose of this paper is twofold. First, a generalization of the ansätze $\mathscr{A}_n$ is described. These new $\mathscr{A}_n$ generate, for each $n \geq 1$, a family of solutions of the self-duality equations depending on two free functions of three variables each. After a general description of the "twistor" construction in Sect. 2, the new $\mathscr{A}_n$ are presented in Sect. 3. There is also some discussion of the problem of how to ensure that a gauge field generated by $\mathscr{A}_n$ is smooth and real-valued, i.e. taking values in the Lie algebra of $SU(2)$ rather than that of $SL(2, \mathbb{C})$.

Section 4 brings us to the second topic of the paper. The multi-monopole solutions referred to above are all superimposed, axially-symmetric configu-

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