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Rigid Curves at Random Positions and Linking Numbers

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Abstract. A property of the square of the linking number of two closed rigid curves randomly displaced in a three dimensional space, has been recently found by W. Pohl. Here, this result is reproduced and generalized. This new approach is quite different and uses a simple Fourier transformation.

1. Introduction

The theory of knots is not only very interesting from a mathematical point of view, it may also have fruitful applications in polymer physics.

A polymer molecule consists of a long sequence of chemically connected units, each of which comprises only a few atoms. The long chains so formed can be quite flexible and for many purposes their behaviour can be modelled by representing them simply by smooth mathematical curves embedded in space (i.e. R^3 or S^3). For instance consider two polymer molecules each of which has the connectivity of a loop (i.e. S^1). In their physical motion, they may move and deform in a continuous fashion but not cross through each other or themselves. At the level of mathematical curves of zero thickness, this is equivalent to saying that they may not pass through configurations with double points. Thus the two molecules conserve their topology and they can physically be separated if and only if their configuration is not topologically linked.

No prescription can be given for classifying completely the linking of two loops. A very simple and versatile partial description is given by Gauss' "linking number" (or "winding number"). Roughly, this quantity is the number of turns one curve winds around the other and vice versa. More precisely we must first define a sense of direction around each of the two curves. Then choose an oriented surface whose boundary is one curve and count algebraically the total number of intersections of the second curve with this surface, ± 1 according to whether the surface is approached from its $\pm$ side by the second curve at each intersection.

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