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Moments of the Auto-Correlation Function and the KMS-Condition

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Abstract. Inequalities between successive moments of the time dependent auto-correlation function are derived. Furthermore, we prove that they provide an infinite set of characterizations of an equilibrium state.

I. Introduction

As is well known, the time auto-correlation function is an important quantity for the study of macroscopic systems both from a theoretical as well as from an experimental point of view. In particular we have in mind response theory. In practice, computations in this theory are generally stopped after the first moment because of lack of information about the higher moments of the auto-correlation function. If this is the case, one speaks about linear response theory [1–4]. Indeed it is not always easy to establish a radius of convergence for this auto-correlation function expressed as a power series in the time variable [5]. Motivated by this problem, we study in a rigorous way the relation between successive terms of this series expansion for KMS-states and derive inequalities between them (see Theorem II.4 and 5). Furthermore we prove that our inequalities are best possible in the sense that each of them characterizes equilibrium or KMS-states (see Theorem III.2 and 3).

First we introduce the scheme in which we work. Let $\mathscr{M}$ be a von Neumann algebra on a Hilbert space $\mathscr{H}$ and H a self-adjoint operator on $\mathscr{H}$. Let $U_t = \exp itH$, $t \in \mathbb{R}$ such that $t \to \alpha_t = U_t \cdot U_t^*$ is a continuous one parameter group of *-automorphisms of $\mathscr{M}$. Let Ω be a normalized element of $\mathscr{H}$ cyclic for $\mathscr{M}$. We denote by ω the vector state determined by Ω.

For any $f \in \mathscr{L}^1(\mathbb{R})$, denote by $\hat{f}$ the Fourier transform $\int dx f(x)\, e^{ikx} = \hat{f}(k)$ of f. Denote by $\mathscr{M}_0$ the algebra generated by the set

$$\{x(f) \in \mathscr{M} | x(f) = \int dt f(t)\, \alpha_t x,\, x \in \mathscr{M}, \hat{f} \in \mathscr{D}\},$$

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