

The Baker Transformation and a Mapping Associated to the Restricted Three Body Problem

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Abstract. The nonlinear mapping of the plane

$$\begin{aligned}x_1 &= x_0 + 1/y_0 \\ y_1 &= y_0 - x_0 - 1/y_0\end{aligned}$$

was recently introduced by Hénon as an asymptotic form of the equations of motion of the restricted three body problem. This is an area preserving diffeomorphism, except along the x -axis where the mapping is singular. We show that this mapping exhibits a type of stochastic behavior known as topological transitivity, by showing that it is topologically conjugate to the well known baker transformation. Consequently, periodic points are dense in the plane and there is also a dense orbit. We note that the baker transformation also preserves Lebesgue measure and is ergodic, so this raises interesting open questions about the ergodic properties of the nonlinear mapping.

Consider the nonlinear mapping of the plane $(x_1, y_1) = F(x_0, y_0)$ given by

$$\begin{aligned}x_1 &= x_0 + \frac{1}{y_0} \\ y_1 &= y_0 - x_0 - \frac{1}{y_0}.\end{aligned}$$

This mapping was recently encountered by Hénon [2] in his studies of the restricted three body problem of classical mechanics. Roughly speaking, this mapping is an asymptotic form of the equations of motion which is related to

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