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A Model Problem with the Coexistence of Stochastic and Integrable Behaviour

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Abstract. A one parameter family of piecewise linear measure preserving transformations of a torus which can be viewed as a perturbation of the twist mapping is introduced. Theorems on their ergodic properties for an infinite set of parameters are proved. For some parameters coexistence of stochastic and integrable behaviour is obtained.

Introduction

The celebrated theorem of Kolmogorov (KAM theory) on the conservation of quasiperiodic motions in classical dynamical systems gave rise to the problem of ergodic properties in the regions where invariant tori are destroyed. Since that time no examples were provided in which the situation could be clarified. Katok [1] gave a construction of a smooth Bernoulli diffeomorphism on the two-dimensional disc which is equal to the identity on the boundary. Hence this transformation can be sewed together with whatever one likes. It proves that in principle quasiperiodic motions and the Bernoulli component can coexist in a smooth dynamical system.

In this paper we give an example of a one parameter family of transformations of a two-dimensional torus for which the coexistence of stochastic and integrable behaviour can be proved for an infinite set of parameters. All transformations are piecewise linear (with two pieces) and hence are not smooth. This family can be viewed as a perturbation of an integrable system – the twist mapping (considered on the torus and not on an annulus as usual). So our example is to some extent given a priori.

The technique used for proving stochastic behaviour is much the same as in [2]. By stochastic behaviour we mean almost hyperbolicity: the existence almost everywhere (in some domain) of local expanding and contracting fibres which form absolutely continuous foliations. By classical methods this allows us to prove that the domain consists of sets of positive measure on which some power of the transformation is a K-system. Actually in our case the methods of Ornstein can be

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