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How Conclusive is the Scaling Argument? The Connection Between Local and Global Scale Variations of Finite Action Solutions of Classical Euler–Lagrange Equations

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Abstract. We analyze the argument that a critical point of the action is stationary under a global scale transformation. We establish a general criterion which allows one to prove rigorously the validity or nonvalidity of the argument in the various relevant classes of Euler–Lagrange equations. Furthermore, we give a priori estimates on solutions at infinity.

1. Introduction

In the physical literature on finite energy solutions (respectively Gibbs-, free energy-, action- etc.) of nonlinear partial differential equations one finds frequently the argument that non-stationarity of the action under global dilations entails the nonexistence of finite action solutions for a wide class of model Lagrangians and for the interesting space dimensions. In other cases global stationarity serves as a means to establish a priori constraints on the solutions. The former point was for the first time emphasized by Derrick [1]. Very readable accounts of the whole subject are [2], [3]. The latter point was exploited in [4] to show the existence of solutions for a wide class of scalar models.

That the so-called "Derrick argument" might perhaps not be fully satisfactory was, as far as we know, for the first time emphasized in [5] for the nonlinear σ-model. In this paper we want to discuss, among other things, the limits respectively validity of this argument in full generality. We then apply the results to several classes of model Lagrangians, including the ones discussed in [4] and some of the models of classical nonabelian gauge theory, e.g. the Prasad–Sommerfield(PS) monopole solution etc.

[1]The critical point in the usual argument is the following. The Euler–Lagrange

1 The notion critical point for a stationary point of the action was a suggestion of Prof. A. Jaffe

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