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The Existence of Multi-Monopole Solutions to the Non-Abelian, Yang–Mills–Higgs Equations for Arbitrary Simple Gauge Groups

Clifford Henry Taubes* †

Department of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. We prove that for arbitrary simple gauge groups, the non-Abelian Yang–Mills–Higgs Equations on $\mathbb{R}^3$ in the Prasad–Sommerfield limit have at least a countably infinite set of distinct solutions. These solutions may be interpreted physically as configurations of widely spaced, non-interacting fundamental monopoles. The solutions are generically not spherically symmetric.

1. Introduction

There is an interest in smooth, classical solutions to non-Abelian, Yang–Mills–Higgs equations on Euclidean $\mathbb{R}^3$. The finite action solutions are solitions in a 4-dimensional, Minkowski space theory with spontaneous symmetry breaking; they are known as magnetic monopoles [1]. Until recently, the only solutions resulted from imposing a spherically symmetric ansatz of one form or another [2, 3, 4]. In this paper, we show that if the gauge group is compact, simply connected and simple (with the Higgs scalars in the adjoint representation) and the Prasad–Sommerfield [5] limit of vanishing scalar field self-interaction is taken, there is at least a countably infinite set of distinct solutions. These solutions are generically, not spherically symmetric. They may be interpreted as configurations consisting of arbitrary numbers of the spherically symmetric monopoles previously mentioned.

In a recent paper (see Chap. IV, [6], the author established sufficient conditions for existence of finite action solutions to the Yang–Mills–Higgs equations in the Prasad–Sommerfield limit. These criteria were applied (in [6.IV]) in the case where the gauge group is $SU(2)$ to produce an existence proof for multimonopoles in this $SU(2)$ case. In this paper we apply the criteria of [6.IV] to the general case

* Harvard University, John Parker Fellow

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