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The Boost Problem in General Relativity

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Abstract. We show that any asymptotically flat initial data for the Einstein field equations have a development which includes complete spacelike surfaces boosted relative to the initial surface. Furthermore, the asymptotic fall off is preserved along these boosted surfaces and there exists a global system of harmonic coordinates on such a development. We also extend former results on global solutions of the constraint equations. By virtue of this extension, the constraint and evolution parts of the problem fit together exactly. Several theorems are given which concern the behaviour in the large of general classes of linear and quasilinear differential systems. This paper contains in addition a systematic exposition of the functional spaces employed.

1. Introduction

Lichnerowicz showed [1] that the Einstein equations form a system in involution and that the Cauchy problem in general relativity consists of two problems: the problem of the initial value constraints and the problem of evolution. Choquet-Bruhat [2] employed the harmonic coordinate condition to reduce the evolution equations to a hyperbolic system and showed the existence and uniqueness of local-in-time solutions. An improvement of this result was later obtained in [3] based on the work of Leray [4] and Dionne [5] on hyperbolic systems. Global uniqueness, namely existence of a unique maximal development of the initial data, was shown by Choquet-Bruhat and Geroch in [6]. The evolution equations were studied from the point of view of a first order symmetric hyperbolic system by Fischer and Marsden [7]. The sharpest differentiability results for the evolution equations are due to Hughes, Kato and Marsden [8]. The problem of the initial value constraints, formulated in a particularly satisfactory way in the work of York [9] and O'Murchadha and York [10], was studied on a compact manifold by Choquet-Bruhat [11], and on a manifold which is euclidean at infinity by Cantor [12, 13] and Chaljub-Simon and Choquet-Bruhat [14].

The existence of local-in-time solutions to the Einstein equations is a result which is perhaps sufficient for the study of cosmology. It is however insufficient for

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