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New Approach to the Semiclassical Limit of Quantum Mechanics

I. Multiple Tunnelings in One Dimension

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Abstract. We propose a new approach for the estimate of the rate of degeneracy of the lowest eigenvalues of the Schrödinger operator in the presence of tunneling based on the theory of diffusion processes. Our method provides lower and upper bounds for the energy splittings and the rates of localization of the wave functions and enables us to discuss cases which, as far as we know, have never been treated rigorously in the literature. In particular we give an analysis of the effect on eigenvalues and eigenfunctions of localized deformations of 1) symmetric double well potentials 2) potentials periodic and symmetric over a finite interval. Theses situations are characterized by a remarkable dependence on such deformations. Our probabilistic techniques are inspired by the theory of small random perturbations of dynamical systems.

1. Introduction

The estimate of the semiclassical rate of degeneracy of the lowest eigenvalues of the Schrödinger operator H in the presence of tunneling is not a new problem and has been solved in special situations, for example in connection with the theory of phase transitions in statistical mechanics [1]. More recently Harrell has produced two papers [2, 3] in which a rather complete analysis of the above problem for the case of symmetric double wells is given and where one can find a wide list of references. The methods employed in these papers require in general a detailed analysis of the eigenfunctions of H as $\hbar \to 0$ and their generalization to non-symmetric cases does not appear so easy. Here we propose a different approach based on the theory of diffusion processes which requires only an estimate of the

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