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Dilation Analyticity in Constant Electric Field

II. N-Body Problem, Borel Summability

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Abstract. We extend the analysis of Paper I from two body dilation analytic systems in constant electric field to N-body systems in constant electric field. Particular attention is paid to what happens to isolated eigenvalues of an atomic or molecular system in zero field when the field is turned on. We prove that the corresponding eigenvalue of the complex scaled Hamiltonian is stable and becomes a resonance. We study analyticity properties of the levels as a function of the field and also Borel summability.

1. Introduction

Our goals in this paper are to extend the formalism developed by Herbst [16] (which describes complex scaling in the presence of constant electric field) from two body systems to N-body systems, to recover the beautiful results of Graffi and Grecchi [13] on Borel Summability of the hydrogen Stark problem within this framework and to extend these summability results to multielectron atoms. Some of our results were announced in [17]. Subsequently, Graffi and Grecchi [37] developed a different formalism which allows a discussion of certain N-body systems in electric field. Their analysis appears to require that all particles have charges with the same sign. Moreover, their method does not reduce to ordinary complex scaling when the electric field is absent. However, since they need only treat *strictly* sectorial operators, their method has some technical advantage over ours.

While it is probable that with some extra effort, we could handle quadratic form perturbations and non-local potentials, we will use the operator class of Aguilar–Balslev-Combes [2, 8] and restrict to local potentials. As usual, let $\mathfrak{H}=L^2(\mathbb{R}^\nu)$, $t=-\Delta$, $(u(\theta)f)(r)=e^{\nu\theta/2}f(e^\theta r)$ for θ real and $\mathfrak{H}_{+1}=D(t)$ with the graph norm. Notice that $u(\theta)$ is bounded from $\mathfrak{H}_{+1}$ to itself.

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