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Yang-Mills-Higgs Monopole Solutions of Arbitrary Topological Charge

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Abstract. We propose a construction of static magnetic Yang-Mills-Higgs monopole solutions of arbitrary topological charge. They are axially symmetric and contain no free parameters except for their position. The regularity of the solutions has yet be proved; doing so would complete the constructive proof of existence.

I. Introduction

The purpose of this paper is to present new static magnetic Yang-Mills-Higgs monopole solutions of arbitrary topological charge. The solutions are axially symmetric and contain no free parameters except for their position in three dimensional Euclidean space. Our solution thus generalizes the recent and remarkable construction of Ward [1] on the charge 2 monopole. Unlike Ward's construction, our construction is based on a systematic framework for obtaining monopole solutions with arbitrary topological charge. As a byproduct of the systematic framework, we are able to explicitly construct the complex gauge transformation that makes Ward's (and our) solution real. In a separate paper [9] we have verified regularity of these solutions in various regions, but we have not yet shown that these regions cover the three dimensional Euclidean space.

Let us define in four dimensional Euclidean space (x_1, x_2, x_3, x_4) the gauge potentials A^a_μ where $a=1,2,3$ and $\mu=1,2,3,4$. The gauge field strength is defined by

$$F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\varepsilon^{abc} A^b_\mu A^c_\nu, \tag{1.1}$$

where e is an arbitrary constant, the gauge coupling constant. The problem, simply stated, is to solve the self duality equations:

$$F^a_{\mu\nu} = +1/2\varepsilon_{\mu\nu\lambda\varrho} F^a_{\lambda\varrho} \tag{1.2}$$

(our convention is $\varepsilon_{1234} \equiv +1$) for the gauge potentials A^a_μ subject to the following requirements:

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