Commun. Math. Phys. 80, 127–136 (1981)

Communications in Mathematical Physics

Some Aspects of the Theory of Defects of Ordered Media and Gauge Fields Related to Foliations

V. Poénaru

Université Paris-Sud, Bâtiment mathématique 425, F-91405 Orsay, France

Abstract. When translational symmetry is broken in the ground state, the homotopy theory of defects of ordered media has to be supplemented with integrability conditions, coming from the theory of foliations. These show how some homotopy classes split into several distinct defects, while other homotopy classes do not occur physically. This framework can also be used in order to discuss defects of gauge fields, where in a first approximation classifying spaces play the role of the manifolds of internal states.

1. Introduction

During the last few years, the topological theory of defects of ordered media has become a relatively well established subject.

The general framework for this approach, as laid down by Toulouse and Kléman, has been, so far, the following. For each type of order, one has a "manifold of internal states", V, characteristic for the order in question.

The mathematical model for a specific ordered medium consists of a physical space M (which is assumed to be an n-dimensional manifold), of a subset of "defect points" $\Sigma \subset M$, and of a continuously defined "order parameter"

$$M - \Sigma \ni p \mapsto \Phi(p) \in V.$$

The standard approach has been to classify defects according to the homotopy classes of the maps Φ or to study the interaction of defects via the algebraic structure of the homotopy group $\pi_i V$ and their Whitehead products (see, for example [20, 7, 15, 16, 22]).

General references for all this are Mermin's review article [9], our Les Houches lectures [17], or the review article of Michel [10].

Limitations of this approach, in the case of ordered media with broken translational symmetry in the uniform state, are very carefully explained in one of the paragraphs of Mermin's article.

The aim of the present paper is to go one small step beyond pure homotopy theory, towards differential geometry, in this topological study of defects. The new

0010-3616/81/0080/0127/$02.00