Commun. Math. Phys. 80, 59–98 (1981)

Communications in Mathematical Physics

Pointwise Bounds on Eigenfunctions and Wave Packets in N-Body Quantum Systems

V. Lower Bounds and Path Integrals*

R. Carmona[1] and B. Simon[2]

Department of Physics, Princeton University, Princeton, NJ 08544, USA

Abstract. By using Agmon's geodesic ideas to single out particular regions in path space, we obtain optimal lower bounds on the leading behavior for the fall off of the ground state of multiparticle system.

1. Introduction

This paper is a contribution to the large literature on the decay at infinity of eigenvectors of Schrödinger operators, $-\frac{1}{2}\Delta + V$, associated to discrete spectrum [1, 3–5, 9, 12, 15, 17, 22–24, 27, 28, 32, 34, 37, 39, 46–48, 51]. For the *leading* behavior of the *ground state*, φ, our results are definitive in the sense that we will show that:

$$\lim_{|x|\to\infty} -[\log\varphi(x)]/\varrho(x) = 1 \tag{1.1}$$

for an explicit function ϱ and for a large class of potentials, V, including general N-body systems. The upper bounds implicit in (1.1) are not new: for multiparticle systems, they were found in successively more general cases by Mercuriev [37] (three-body), Deift et al. [17], and Hoffman-Ostenhoff et al. [4] (atoms with infinitely heavy nucleus) and Agmon [1] in the general case; for potentials going to infinity at infinity they were found by Lithner [34] and rediscovered by Agmon [1]. The Lithner-Agmon upper bounds are only proven to hold in some average sense, but it is easy to get pointwise bounds with minor extra restrictions on V (see Appendix 2). Our primary goal here will be to find lower bound complementary to these various upper bounds which show that the upper bounds are "best possible".

A major source of motivation for the approach we use is the part of Agmon's work [1] which identifies the function ϱ in (1.1). Let us initially describe the situation for the case $V\to\infty$, $V\geqq 1$ and continuous, a case treated by Lithner, with a related intuition.

* Research partially supported by UNSF Grant MCS-78-01885

1 On leave from: Département de Mathématiques, Université de Saint Etienne, 23 rue P. Michelon, F-42023 Saint Etienne Cédex, France

2 Also at Department of Mathematics, Princeton University

0010-3616/81/0080/0059/$08.00