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Regularized, Continuum Yang-Mills Process and Feynman-Kac Functional Integral

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Abstract. Giving an ultraviolet regularization and volume cut off we construct a nuclear Riemannian structure on the Hilbert manifold $\mathfrak{M}$ of gauge orbits. This permits us to define a regularized Laplace-Beltrami operator Δ on $\mathfrak{M}$ and an associated global diffusion in $\mathfrak{M}$ governed by Δ. This enables us to define, via a Feynman-Kac integral, a Euclidean, continuum regularized Yang-Mills process corresponding to a suitable regularization (of the kinetic term) of the classical Yang-Mills Lagrangian on $T\mathfrak{M}$.

Introduction

In order to make a serious study of non-perturbative quantum Yang-Mills theory in four dimensional space-time, it appears to be indispensable to construct first a regularized Euclidean Yang-Mills theory via a well defined functional integral. This first step has so far only been achieved in the context of lattice gauge theories [21, 11, 19], no rigorous construction in the continuum with cut-offs having yet been given. The serious block to a direct continuum construction appears to have been the nature of the configuration space of Yang-Mills theory which is the space of gauge orbits [16]. This space is not a linear space and in it we have no globally defined notion of a free field. Thus, as should be obvious, we cannot interpret the cut-off Yang-Mills theory as a free field plus regularised perturbation. There exists a formal quantization procedure going back to de Witt, Feynman, Faddeev-Popov [7] which has enjoyed much success in perturbation theory. But this procedure, in its present form even with cut-offs, is fraught with ambiguities at all levels, measure theoretic and geometric, and cannot be considered as a viable alternative to lattice gauge theories for non-perturbative considerations.

In this paper we shall see how some of these problems can be overcome; in fact we shall give a rigorous definition of a global Euclidean Feynman-Kac integral with regularization working directly in the space of gauge orbits. Our construction

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