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The Stark Ladder and Other One-Dimensional External Field Problems

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Abstract. For a certain class of analytic potentials $V(x)$, matrix elements of the resolvent of $H_F = -d^2/dx^2 + Fx + V(x)$ with entire vectors of the translation group have meromorphic continuations from $\operatorname{Im} z > 0$ to the whole complex plane. The poles of these continuations are restricted to a discrete set independent of the analytic vectors chosen. Certain random potentials corresponding to an infinite number of particles distributed on the points of a Poisson set lie in this class with probability one as do a large class of periodic potentials.

1. Introduction

It is believed that when a uniform electric field F is applied to a one-dimensional periodic solid described by

$$H = -d^2/dx^2 + V(x)$$

with $V(x)$ periodic of period 1, each band gives rise to an infinite sequence of resonances of fixed imaginary part, located at

$$E_n = E_0 + nF \quad n = 0, \pm 1, \pm 2, \ldots$$

where $\operatorname{Im} E_0 < 0$. Although the Hamiltonian

$$H_F = -d^2/dx^2 + Fx + V(x)$$

may look rather simple, no rigorous proof of the existence of these resonances has yet been given, to say nothing of the important problem of estimating the lifetime. Attacking the problem from a different point of view, Bentosela [6] has shown, in any number of dimensions, the existence of states ψ such that $e^{-itH_F}\psi$ has a momentum distribution with the momentum nearly periodic in time over many periods $T = 2\pi/F$.

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