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On the Existence of Solutions to Einstein's Equation With Non-Zero Bondi News

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Abstract. It is shown that the C-metric (with parameters chosen to lie in suitable intervals) admits a conformal completion such that the space of generators of null infinity, $\mathscr{I}$, is a 2-sphere. This structure of $\mathscr{I}$ is both necessary and sufficient for the analysis of gravitational radiation in exact general relativity. Bondi news (as well as the electromagnetic radiation field, in the charget case) is examined and found to be non-zero. Thus the issue of existence of exact solutions to the Einstein (and Einstein-Maxwell) equations admitting radiation (in the sense of Bondi, Sachs, and Penrose) is resolved. In addition, the analysis clarifies the sense in which the vacuum C-metric represents the gravitational field of two accelerating black-holes.

1. Introduction

Penrose's [1] null infinity, $\mathscr{I}$, provides an elegant, geometrical method to impose the boundary conditions that are believed to be appropriate in problems involving gravitational radiation. Furthermore, the single assumption that a given space-time admits a $\mathscr{I}$ whose space of generators is a 2-sphere enables one to obtain a rich structure of direct physical significance[1]: one can introduce, in a precise fashion, various asymptotic fields, prove peeling theorems, show that the flux of energy carried away by gravitational waves is positive, compute the Bondi 4-momentum at any retarded instant of time, etc. Consequently, over the years, the notion of $\mathscr{I}$ has become crucial to the gravitational radiation theory: essentially every rigorous result conerning radiation in exact general relativity assumes, at least implicitly, the existence of $\mathscr{I}$. Furthermore, without recourse to $\mathscr{I}$, one can

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1 If the space of generators fails to be a 2-sphere, however, one can do very little: one can introduce neither the news tensor nor the Bondi 4-momentum. For details, see e.g. [1] or [2]