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# Tetrahedron Equations and the Relativistic $S$-Matrix of Straight-Strings in $2+1$-Dimensions

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**Abstract.** The quantum $S$-matrix theory of straight-strings (infinite one-dimensional objects like straight domain walls) in $2+1$-dimensions is considered. The $S$-matrix is supposed to be "purely elastic" and factorized. The tetrahedron equations (which are the factorization conditions) are investigated for the special "two-colour" model. The relativistic three-string $S$-matrix, which apparently satisfies this tetrahedron equation, is proposed.

## 1. Introduction

The progress of the last decade in studying two-dimensional exactly solvable models of quantum field theory and lattice statistical physics was motivated to some extent by using *the triangle equations*. These equations were first discovered by Yang [1]; they appeared in the problem of non-relativistic $1+1$-dimensional particles with $\delta$-function interaction, as the self-consistency condition for Bethe's ansatz. Analogous (at least formally) relations were derived by Baxter [2], who had investigated the eight-vertex lattice model. These relations restrict the vertex weights and are of great importance for exact solvability. In particular, for the rectangular-lattice model they guarantee the commutativity of transfer-matrices with different values of the anisotropy parameter $v$. In the case of Baxter's general nonregular lattice $\mathscr{L}$ [3], the triangle relations for the vertex weights ensure the remarkable symmetry of the statistical system (the so-called $Z$-invariance): the partition function is unchanged under the deformations of the lattice, generated by the arbitrary shifts of the lattice axes. $Z$-invariant model on the lattice $\mathscr{L}$ is exactly solvable [3] (see also [4]).

Recently Faddeev, Sklyanin, and Takhtadjyan [5, 6] have developed a new general method of studying the exactly solvable models in $1+1$-dimensions – the quantum inverse scattering method. The triangle equations are the significant constituent of this method; they are to be satisfied by the elements of the $R$-matrix which determine the commutation relations between the elements of the monodromy matrix.

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