Commun. Math. Phys. 79, 473–488 (1981)

Communications in Mathematical Physics

Two-Dimensional Generalized Toda Lattice

A. V. Mikhailov[1], M. A. Olshanetsky[2], and A. M. Perelomov[2]

1 Landau Institute of Theoretical Physics USSR
2 Institute of Theoretical and Experimental Physics, SU-117259 Moscow, USSR

Abstract. The zero curvature representation is obtained for the two-dimensional generalized Toda lattices connected with semisimple Lie algebras. The reduction group and conservation laws are found and the mass spectrum is calculated.

1. Introduction

In recent work [1] it was shown that the two-dimensional generalization of the classical periodic Toda lattice (TL) is solved by the inverse scattering method and the reduction from the complete Zakharov-Shabat equations was found. On the other hand, Bogoyavlensky constructed the generalized TL connected with the root systems of the semisimple Lie algebras [2]; the classical TL then corresponds to the root system of the type $A_{\ell-1}$. The purpose of the present paper is to generalize the results obtained in [1] on arbitrary root systems, in other words, to give a two-dimensialization of the lattices constructed in [2]. This generalization has some new features when compared with the system of type $A_{\ell-1}$ [1]. The results obtained are given in the most general form possible that enables one to understand the invariant meaning of the formulae in [1].

The plan of the paper is as follows: in Sect. 2 we describe the generalized TL and give a brief introduction to systems of roots. In Sect. 3 we construct a reduction group from a complete Lie algebra. In Sect. 4 we compute the mass spectrum of our systems and in Sect. 5 we investigate conservation laws.

2. The Description of the Systems

We shall investigate the relativistic systems with Lagrangians

$$L=\sum_{k=1}^{\ell}\partial_\mu\varphi^k\partial^\mu\varphi^k-\tfrac{1}{2}U(\varphi^1,\ldots,\varphi^\ell)$$
$$\partial_\mu=\partial/\partial x_\mu \qquad \mu=0,1\,, \tag{2.1}$$

0010-3616/81/0079/0473/$03.20