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On the Bundle of Connections and the Gauge Orbit Manifold in Yang-Mills Theory

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Abstract. In an appropriate mathematical framework we supply a simple proof that the quotienting of the space of connections by the group of gauge transformations (in Yang-Mills theory) is a C^∞ principal fibration. The underlying quotient space, the gauge orbit space, is seen explicitly to be a C^∞ manifold modelled on a Hilbert space.

0. Introduction

In [1], Singer announced interesting results on the quotienting of the space of C^∞ connections of a principal G-bundle [on compact orientable Riemannian space without boundary by the group of gauge transformations under appropriate restrictions (essentially free group action)]. In particular [1], the quotienting is a principal fibration, and the underlying quotient space (gauge orbit space) is C^∞ manifold. In [2] Narasimhan and Ramadas prove independently that the quotienting in question is a principal fibration (for Sobolev spaces of connections). In [1, 2] it is proved that when $G=SU(N)$, and the initial base space S^d $(d=3,4)$, the corresponding fibration is nontrivial. The gauge orbit space is not contractible. Thus continuous global gauge fixing (section) is not possible.

These global results are of relevance to quantum gauge field theory where the dynamical variables are supplied by the gauge orbit space.

The present paper is motivated by the need, on the part of gauge field theorists, to understand better the geometry of the gauge orbit space, for reasons adduced below. We return to the quotienting of the space of irreducible connections by the group of gauge transformations (restricted to free group action) within the mathematical framework of [2], i.e. we work with Sobolev spaces of sections of various bundles. We prove that the gauge orbit space is a C^∞ manifold modelled on a Hilbert space. In order to prove this directly, and to exhibit the C^∞ structure, we give an alternative proof (to that of [2]), that we have a principal fibration, in fact a C^∞ fibration. Our strategy is to use the existence of local sections to give

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