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The Geometry of Gauge Field Copies*

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To the Memory of Jorge André Swieca

Abstract. We show that a gauge field uniquely determines its potential if and only if its holonomy group coincides with the gauge group on every open set in spacetime, provided that the field is not degenerate as a 2-form over spacetime. In other words, there is no potential ambiguity whenever such a field is irreducible everywhere in spacetime. We then show that the ambiguous potentials for those gauge fields are partitioned into gauge-equivalence classes (modulo certain homotopy classes) as a consequence of the nontrivial connectivity of spacetime. These homotopy classes depend on the gauge group, on the holonomy group and on this last group's centralizer in the gauge group.

1. Introduction

The potential ambiguity for nonabelian gauge fields, that is, the existence of "gauge field copies", has been widely discussed [1] since it was discovered by Wu and Yang [2]. Some gauge fields admit two or more potentials. This potential ambiguity falls into two large classes with a nonvoid intersection: ambiguous potentials are sometimes (but not always) equivalent modulo a gauge transformation. There is also an in-between situation, for some gauge fields admit at each gauge a whole system of potentials which is partly but not completely reducible with the help of gauge mappings [3, 37].

The present paper gives necessary and sufficient conditions for the existence or inexistence of potential ambiguities in a given gauge field provided that the gauge field is not degenerate as a 2-form on spacetime. Potential ambiguity is shown here to be a consequence of too many degrees of freedom in the field. In other words, if we adopt the bundle description [4] for gauge fields, there will be a potential ambiguity whenever the bundle is (at least locally) reducible. When the bundle is maximally twisted, that is, when the holonomy group coincides with the whole gauge group on every open set in the base manifold, we find no copies at all.

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