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On the Large Order Expansion for the Anharmonic Oscillators

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Abstract. Using functional approaches, we investigate the large-K behaviour of the K^{th} coefficient E_K in the perturbation expansion for the ground-state energy $E(g)$ of the generalized anharmonic oscillator X^{2N} with internal $O(n)$-symmetry. We establish the equivalence between the pure functional approach and the method of Collins-Soper at any order in $\frac{1}{K}$. For that purpose, we first develop an algebraic treatment of perturbation series and prove a theorem on Borel-summable functions. Finally, we compute analytically the $1/K$ and $1/K^2$ corrections to the leading term for $N=2$.

I. Introduction

Some years ago, Bender and Wu [1] investigated the large order behaviour of the perturbation expansion of the energy levels of the anharmonic oscillator. More recently, in a series of works initiated by Lipatov [2], functional techniques have been applied to the determination of such behaviours. In the case of the generalized anharmonic oscillator, the quantity of main interest is the ground-state energy

$$E(g) = -\lim_{T\to\infty} \frac{1}{T} \operatorname{Log} \operatorname{Tr} \exp[-TH], \tag{1.1}$$

where

$$H = \tfrac{1}{2}\sum_{i=1}^{n} p_i^2 + \tfrac{1}{2}\sum_{i=1}^{n} x_i^2 + g\left(\sum_{i=1}^{n} x_i^2\right)^N, \tag{1.2}$$

and one looks for the large-K behaviour of the coefficients E_K in the asymptotic expansion

$$E(g) = \sum_{K=0}^{\infty} E_K g^K. \tag{1.3}$$

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