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On the Construction of Quantized Gauge Fields

III. The Two-Dimensional Abelian Higgs Model Without Cutoffs

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Abstract. In this paper the construction of the two-dimensional abelian Higgs model begun in two earlier articles is completed. First we show how to remove the remaining ultraviolet cutoff on the gauge field, then we construct the infinite volume limit and verify the axioms of Osterwalder and Schrader for the expectation values of gauge invariant local fields. Finally it is shown that an auxiliary gauge field mass that was introduced to avoid infrared problems can be safely removed.

1. Introduction and Notation

In this paper we continue our investigation of quantized gauge fields begun in [1, 2] by constructing a cutoff free version of the abelian Higgs model in two dimensions obeying all the Osterwalder-Schrader axioms (except possibly clustering) and therefore corresponding to a Wightman theory. From our study of the theory on the lattice we have reason to believe that this theory in fact does have exponential clustering for gauge invariant observables; this is the well known Higgs mechanism. In order to verify this for the continuum theory, one would have to work harder than we do in the present paper and construct a convergent expansion around some mean field theory in the spirit of [3]: the mean field configurations would presumably be configurations of vertices.

The plan of this paper is as follows: After fixing notation we show stability of the theory in a finite volume by an expansion that is, of course, inspired by earlier work in constructive quantum field theory, in particular [4]. This is done in Sect. 2; some technical matters concerning Feynman graphs are deferred to an Appendix. The difficulty of the problem lies somewhere between the two dimensional Yukawa model and the three-dimensional ϕ^4 theory; the fields (in particular the gauge field) have to be localized only in momentum space, not in phase space. It is important to preserve gauge invariance in the form of the Ward

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