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A Method of Integration over Matrix Variables

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Abstract. The integral over two $n \times n$ hermitan matrices $Z(g,c) = \int dA dB \exp\left\{-\operatorname{tr}\left[A^2 + B^2 - 2cAB + \frac{g}{n}(A^4 + B^4)\right]\right\}$ is evaluated in the limit of large n. For this purpose use is made of the theory of diffusion equation and that of orthogonal polynomials with a non-local weight. The above integral arises in the study of the planar approximation to quantum field theory.

1. Introduction

In their study of planar diagrams some authors [1, 3] have discussed integrals of the form

$$Z = \int \prod_i dM^{(i)} \exp\left\{-\sum_i V(M^{(i)}) + \sum_{i<j} C_{ij} \operatorname{tr} M^{(i)} M^{(j)}\right\} \tag{1.1}$$

$$V(M) = \operatorname{tr} M^2 + \frac{g}{n} \operatorname{tr} M^4 \tag{1.2}$$

where $M^{(1)}, M^{(2)}, \dots$ are hermitian matrices of order $n \times n$. The integral is taken over all independent real parameters entering the matrix elements,

$$\int dM = \int_{-\infty}^{\infty} \dots \int \prod_{i=1}^{n} dM_{ii} \prod_{1 \leqq i < j \leqq n} d(\operatorname{Re} M_{ij}) d(\operatorname{Im} M_{ij}). \tag{1.3}$$

The case of one matrix is the simplest. There are no cross terms containing C_{ij}. The integral reduces to that over the eigenvalues [4],

$$\begin{aligned} Z(g) &= dM \exp\left\{-\operatorname{tr} M^2 - \frac{g}{n} \operatorname{tr} M^4\right\} \\ &= \text{const.} \int \exp\left\{-\sum_{i=1}^{n}\left(x_i^2 + \frac{g}{n} x_i^4\right)\right\} |\Delta(X)|^{\beta} \prod^{n} dx_i, \end{aligned} \tag{1.4}$$

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