Commun. Math. Phys. 79, 317–325 (1981)

Communications in Mathematical Physics

A Yang–Mills–Higgs Monopole of Charge 2

R. S. Ward
Department of Mathematics, Trinity College, Dublin, Ireland

Abstract. A new static, purely magnetic Yang–Mills–Higgs monopole solution is presented. It is axisymmetric and has a topological charge of 2; the charge is located at a single point.

1. Introduction

This paper is concerned with Yang–Mills–Higgs monopoles which are static and purely magnetic, in the Prasad–Sommerfield limit [1]. This means that we have a Higgs field ϕ and a gauge potential A_j ($j = 1, 2, 3$) on Euclidean 3-space $\mathbb{R}^3$, satisfying the following five requirements.

(i) ϕ & A_j take values in the Lie algebra of SU(2). In other words, ϕ and A_j have the form $\phi = \phi^a \sigma^a$, $A_j = A_j^a \sigma^a$, where σ^a are the Pauli matrices, and where ϕ^a, A_j^a are scalar functions on $\mathbb{R}^3$ which in some gauge are real-valued. (We shall be allowing SL(2, $\mathbb{C}$)-valued gauge transformations, so σ^a & A_j^a will not be real-valued in every gauge.)

(ii) In some gauge, ϕ & A_j are smooth (say C^∞) on $\mathbb{R}^3$.

(iii) The Bogomolny equations

$$G_{jk}^a = -\varepsilon_{jk\ell} D_\ell \phi^a$$

are satisfied, where

$$G_{jk}^a = \partial_j A_k^a - \partial_k A_j^a + \kappa \varepsilon^{abc} A_j^b A_k^c,$$

$$D_j \phi^a = \partial_j \phi^a + \kappa \varepsilon^{abc} A_j^b \phi^c,$$

κ being some real number (the coupling constant).

(iv) The norm $\| \phi \| = (\phi^a \phi^a)^{1/2}$ of the Higgs field has the asymptotic behaviour

$$\| \phi \| = 1 - m/r + 0(r^{-2}) \text{ as } r \to \infty,$$

where r is the Euclidean distance from the origin in $\mathbb{R}^3$, and m is some real number.

(v) The energy

$$E = \int (\tfrac{1}{4} \| G \|^2 + \tfrac{1}{2} \| D\phi \|^2) d^3x$$

is finite. Here $\| G \|^2 = G_{jk}^a G_{jk}^a$ and $\| D\phi \|^2 = (D_j \phi^a)(D_j \phi^a)$.

0010-3616/81/0079/0317/$01.80