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Proof of the Positive Mass Theorem. II

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Abstract. The positive mass theorem states that for a nontrivial isolated physical system, the total energy, which includes contributions from both matter and gravitation is positive. This assertion was demonstrated in our previous paper in the important case when the space-time admits a maximal slice. Here this assumption is removed and the general theorem is demonstrated. Abstracts of the results of this paper appeared in [11] and [13].

Introduction

An initial data set for a space-time consists of a three-dimensional manifold N, a positive definite metric g_{ij}, a symmetric tensor p_{ij}, a local mass density μ, and a local current density J^i. The constraint equations which determine N to be a spacelike hypersurface in a space-time with second fundamental form p_{ij} are given by

$$\mu=\tfrac{1}{2}\left[R-\sum_{i,j}p^{ij}p_{ij}+\left(\sum_i p_i^i\right)^2\right]$$

$$J^i=\sum_j D_j\left[p^{ij}-\left(\sum_k p_k^k\right)g^{ij}\right],$$

where R is the scalar curvature of the metric g_{ij}. As usual, we assume that μ and J^i obey the dominant energy condition

$$\mu \geqq \left(\sum_i J^i J_i\right)^{1/2}.$$

An initial data set will be said to be asymptotically flat if for some compact set C, $N\backslash C$ consists of a finite number of components $N_1, \ldots, N_p$ such that each N_i is diffeomorphic to the complement of a compact set in R^3. Under such diffeomorphisms, the metric tensor will be required to be written in the form

$$g_{ij}=\delta_{ij}+O(r^{-1})$$

and the scalar curvature of N will be assumed to be $O(r^{-4})$.

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