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On the Symmetry of the Gibbs States in Two Dimensional Lattice Systems

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Abstract. Under fairly general conditions if a two dimensional classical lattice system has an internal symmetry group G, which is a compact connected Lie group, then all Gibbs states are G-invariant.

1. Introduction

For a large class of classical lattice systems with an internal symmetry described by a continuous group G all Gibbs states are also G-invariant if the space dimension is two [1, 2]. One says that spontaneous symmetry breakdown is impossible. This phenomenon occurs in various other situations. We refer to [3] for examples and rigorous results in the field of statistical mechanics. Results of this kind are established here for classical lattice systems on $\mathbb{Z}^2$ with a compact connected Lie group G. A lattice system is given by a measure space Ω_x, which is the space of configurations of the system at the lattice point x, a measure dw_x on each Ω_x and a potential U describing the interactions in the system. For example $\Omega_x = S^1$, the unit circle, dw_x is the uniform measure on S^1 and U is given by two-body interactions $-J(x-y)\cos(w_x - w_y)$ which are G-invariant with $G = S^1$ in an obvious way. Here $w_x \in \Omega_x$ and $w_y \in \Omega_y$. If $J(x-y) = |x-y|^{-\alpha}$, then the system is ferromagnetic. Theorem 1 below proves that for $\alpha \geqq 4$ all Gibbs states are G-invariant and there is no spontaneous magnetization. On the other hand if $2 < \alpha < 4$ there is spontaneous magnetization at low temperature and therefore there are Gibbs states which are not G-invariant [4]. This remains true with $\Omega_x = S^n$, the n-sphere in $\mathbb{R}^{n+1}$, and $(w_x | w_y)$ instead of $\cos(w_x - w_y)$, where $(-|-)$ is the Euclidean scalar product in $\mathbb{R}^{n+1}$. G is S^n and the results follow from [5] when $2 < \alpha < 4$.

The results of this paper extend previous results obtained by Dobrushin and Shlosman [1] and [2]. First of all the theorem below covers the case of power-law decaying interactions and not only exponentially decaying interactions (see also Remark 2 at the end of Sect. 2). This extension gives complete results for the class of ferromagnetic systems introduced above (see also Remark 1 at the end of

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