Commun. Math. Phys. 79, 153–165 (1981)

Communications in
Mathematical
Physics

The Classical Field Limit of $P(\varphi)_2$ Quantum Field Theory

Matthew Donald*
White Hall, Cornell University, Ithaca, NY 14850, USA

Abstract. It will be shown that, for a convex polynomial P, the $P(\varphi)_2$ quantum field theory without cutoff has a classical field limit as Planck's constant h tends to zero. This extends work of Hepp [1], who considered theories with a space cutoff.

0. Introduction

The purpose of this paper is to show that, for suitable interactions P, the two-dimensional $P(\varphi)_2$ models have a limit, as Planck's constant h tends to zero, which describes a classical field theory. The framework in which this result is proved was first formulated by Hepp [1], who proved a similar result for models with a space cutoff. In the present case, without any cutoff, the technical details are much harder, in particular because the physical Hilbert space Ξ_{ph} of the interacting theory turns out to depend on h.

The central idea is to define a vector $\Psi(h, u^0, v^0) \in \Xi_{ph}(h)$, which depends on initial conditions u^0, v^0 for the classical field theory (see Theorem 1), and then to prove that the expectation value at any time t, of a bounded function of the time zero field tends as h tends to zero, to the same function of the classical field at time t with the initial conditions u^0, v^0. The method, which is also the underlying method in Hepp's paper, is to make a change of variable in the physical Hilbert space, a space of fields, to centre around the classical field, instead of around the zero field. This change takes Ψ to the vacuum vector, and transforms the Hamiltonian into a time dependent Hamiltonian (see Lemma 2). To make this formalism rigorous, a space cutoff ℓ is introduced, and then the problem is to define and control the time dependent Hamiltonian uniformly in h and ℓ. The definition of the time dependent Hamiltonian is by the method of time dependent quadratic forms [9] and the control is by cluster expansion techniques [8], the latter requiring an analytic continuation in an appropriate, but quite unphysical, parameter to a Euclidean region.

It should be noted, that just as there are classical field states, the vectors $\Psi(h, u^0, v^0)$, so also are there classical particle states in the $P(\phi)_2$ models [12].

---

* Present address: Rockefeller University, New York, NY 10021, USA

0010-3616/81/0079/0153/$02.60