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Gauge Dependence of World Lines and Invariance of the S-Matrix in Relativistic Classical Mechanics*

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Abstract. The notion of world lines is studied in the constraint Hamiltonian formulation of relativistic point particle dynamics. The particle world lines are shown to depend in general (in the presence of interaction) on the choice of the equal-time hyperplane (the only exception being the elastic scattering of rigid balls). However, the relative motion of a two-particle system and the (classical) S-matrix are independent of this choice.

Introduction

We study the notion of particle world lines in the relativistic phase space formulation of classical point particle dynamics developed in [19] on the basis of Dirac's theory of constraint Hamiltonian systems[1] [4, 6, 7].

Aiming at a manifestly covariant picture we start with a $8N$-dimensional "large N-particle phase space" Γ^N equipped with a canonical Poisson bracket structure. The dynamics is specified by the introduction of a $7N$-dimensional Poincaré invariant submanifold $\mathscr{M}$ of Γ^N, called the generalized (N particle) mass shell. It is

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1 The constraint Hamiltonian approach to the relativistic point particle interaction was also adopted (in fact, rediscovered) in [9]. Recent work by Rohrlich [15], which proceeds on similar lines, differs from ours in that it abandons the notion of individual particle co-ordinates and trajectory (a generalized notion of "relative co-ordinates" – whose sum over all particles is not required to vanish – is used instead). As noted by Prof. Rohrlich (private communication of October 1978) this difference is not essential: a slight modification of his approach allows one to impose a linear relation among the relative co-ordinates ξ of [15] and hence to define single particle's co-ordinates. A Lagrangian approach to the problem of relativistic point particle interactions, which leads to similar constraint equations is being developed in the work of Takabayasi et al. (see [17] and further references cited therein). The work of Droz-Vincent [5], Bel, Martin [1] and others follows a similar path; their approach differs from ours by introducing from the outset non-canonical position variables (defined only implicitly in terms of the canonical four-dimensional co-ordinates and momenta used in this paper)

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